Trisduction **Series** ·
Preprint

METHODOLOGICAL THESIS

The Uncanny Efficacy and the Unforgiving Limits of Mathematics in Mapping to Reality

'Execution and Admission: Why an Architecture Can Compute Only Graduate Mathematics and Still Require Research-Tier Theorems That Touch No Number'

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ABSTRACT

A verification architecture called Trisduction performs every one of its computations at or below the level of graduate numerical linear algebra. That fact invites an inference: shallow mathematics, therefore weak claims. This paper shows the inference unsound and the unsoundness structural rather than a defect of the survey. The word *difficulty* carries three separable quantities. The first is the marginal cost of executing a competence on machinery already entrenched in the executor. The second is the length of the minimal formal apparatus that models it. The third is the office a piece of mathematics performs on a conclusion, meaning what it does mechanically to that conclusion's standing. These come apart, and only the first is what a difficulty ranking measures. A difficulty level is moreover an output of a chart-selection function, exhibited by one unchanged corpus carrying two level numbers under a reorganized curriculum, and no finer survey converts it into a fact about mathematics. Applying an office taxonomy of five functions, decided by a deletion test run on the citation rather than on the proposition, the architecture resolves into two spines. An execution spine computes, tops out at graduate level, and contains no research-tier object whatever, where *research-tier* names not a difficulty band but a status: every object that appears in no computation at all. An admission spine governs what a result may claim, computes nothing, and cannot operate without research-tier arguments and undergraduate linguistic screens conjoined. A fifth inventory status, Routed, records the difference between an object a corpus never names and an object a corpus never covers, which are not the same thing. Three falsifiers are pre-registered and re-executable at a stated seed. The paper assumes no prior knowledge of the architecture, builds every term it uses, and claims no new mathematics.

KEYWORDS philosophy of mathematics, verification, mathematical depth, epistemic method, Kolmogorov complexity, proof-theoretic strength, invariant theory, quaternion algebra, orientation blindness

1. THE QUESTION AND WHY IT IS NOT THE OBVIOUS ONE

A complete census of the mathematics that the Trisduction verification architecture actually executes returns a short list. It forms correlation matrices of three rows over a modest number of contexts. It removes shared influences by orthogonal projection. It takes a three-by-three determinant. It multiplies three quaternions and reads the real part of the product. It splits a four-by-four reflection into its eigenspaces. It cross-checks the determinant by four disjoint algorithms, reads a condition number against a threshold, calibrates a correlation against two null distributions, and resamples to test stability. Nothing on that list exceeds what a competent graduate student in applied mathematics performs without ceremony.

Set that census beside any canonical ranking of mathematical difficulty and the comparison worsens. On one widely circulated list of the discipline's hardest material, six items are named nowhere in the architecture's corpus under those names: motivic cohomology, the Langlands functoriality conjecture, quantum groups, the local and micro-local analysis of large finite groups, superstring theory, and non-abelian reciprocity with its automorphic representations and modular varieties. Of the remaining four, advanced number theory, large cardinals, and algebraic topology appear only as citations and never as computations, and infinite-dimensional function spaces are touched once, in a clause declaring the object load-bearing on nothing in the same sentence that introduces it.

The inference is immediate and it is what this paper exists to block. It runs: the architecture executes only graduate mathematics, therefore it is mathematically shallow, therefore its claims to structural necessity are decorative. Each step looks sound. The chain is not, and no better inventory repairs it.

The failure is that *difficulty* is doing the work of three quantities that come apart under exactly the operation the argument performs on them. This paper separates them, supplies the one the existing literature does not carry, gives it a mechanical test, and applies all three to one architecture read end to end.

The paper assumes no acquaintance with that architecture. Sections 2 through 5 build the setting: how the relation between mathematics and nature has been understood historically, what account of mathematics this architecture holds, why the classical instruments of inference each leave one thing uncertified, and what the architecture actually is. Section 6 states the role mathematics plays inside it and the surprising order that role forced. Sections 7 through 10 carry the argument: the three quantities, the inventory, the two spines, and where the architecture's counts come from. Section 11 reports the executed figures, section 12 the falsifiable predictions, section 13 the discussion and the limits, and section 14 two structural remarks with the inflated readings of each fenced off. A reader who wants only the result may begin at section 7; a reader who wants to know why the result is not trivial will want the road.

2. MATHEMATICS AND NATURE: THE HISTORICAL POSITIONS

The question of why mathematics fits the world is old, and the answers fall into a small number of families. Naming them is not decoration here. The account developed in section 3 is intelligible only as a departure from them, and the epistemic argument of section 4 turns on a defect all of them share.

The Pythagorean and Platonic family. The oldest answer is that mathematics fits the world because the world is mathematical. Number is the substance of things for the Pythagoreans; for Plato the objects of geometry are more real than their physical shadows, and the physical world participates in them. The position survives in full strength today, as mathematical Platonism, which holds that mathematical objects exist abstractly and mind-independently, and as ontic structural realism, which holds that physical structure simply is mathematical structure.

The family's strength is that it explains the fit completely. Its cost is an access problem, sharpened by Benacerraf. If mathematical objects are causally inert and outside space and time, no causal account of how we come to know them is available. Every proposed bridge then either gives the objects causal powers, which contradicts the thesis, or gives knowledge a non-causal source, which is the thing that wanted explaining.

The Aristotelian and empiricist family. Mathematics is abstracted from the world rather than prior to it. Aristotle takes mathematical properties to be features of physical substances considered apart from their matter; Mill takes arithmetic to be very well-confirmed empirical generalization. The family removes the access problem at a stroke, since abstraction from experience needs no special faculty. Its cost is necessity: it cannot easily explain why arithmetic seems to

hold in every possible world rather than merely in this one, and why mathematics developed with no application in view so often finds one later.

The Kantian family. Mathematics fits appearances because it articulates the form appearances must take for a mind like ours. Space and time are forms of intuition, and geometry and arithmetic are synthetic a priori. The position elegantly explains both necessity and applicability, at the price of restricting the fit to appearance rather than to nature itself. Its historical difficulty is well known: the discovery of consistent non-Euclidean geometries and their physical realization in general relativity removed the specific content Kant assigned to spatial intuition, though the general shape of the position survives in various naturalized forms.

The formalist and conventionalist family. Mathematics is the manipulation of symbols under rules, or a network of conventions convenient for description. Hilbert's program aimed to secure the whole of mathematics by finitary consistency proofs; Poincaré and later Carnap treated substantial parts of the framework as conventional. This family has no access problem and no necessity problem, since there is nothing to access and nothing is necessary in the relevant sense. Its cost is the fit itself: if mathematics is a game, its repeated and unforced success in physical prediction is unexplained, and Gödel's incompleteness theorems removed the specific formalist hope that consistency could be secured from inside.

The unreasonable-effectiveness position. Wigner's 1960 essay named the puzzle rather than solving it: the applicability of mathematics to physics is a wonderful gift we neither understand nor deserve. Hamming's 1980 reply pressed four partial explanations and judged none sufficient. The exchange is the honest modern statement of the problem, and it is honest precisely because it refuses to close.

The selection and anthropic replies. A family of responses holds that the puzzle dissolves under selection effects. We notice the mathematics that works and forget what did not; we build instruments that measure what our mathematics can express; evolution shaped cognition to track regularities that are, by construction, the ones a compressed description captures. These replies are strong against the naive form of the puzzle and weaker against its sharpened form, since they explain why *some* mathematics fits without explaining why fits of extraordinary precision, unavailable at the time of the mathematics' invention, keep occurring.

The maximalist reply. Tegmark's mathematical universe hypothesis takes the Platonic answer to its limit: physical existence and mathematical existence are the same thing, and every mathematical structure is physically real. The fit is then trivial. The cost is a measure problem so severe that the hypothesis makes no differential predictions, and the position is generally treated as a metaphysical stance rather than a scientific one.

What the families share. Each family answers a question of the form *why does mathematics fit*, and each treats the fit as a single relation with a single explanation. None asks a prior question: *what does a given piece of mathematics do* in a particular argument. That question is not about fit at all. It is about function, it has different answers for different citations of the same theorem, and no answer to the fit question determines it. Section 7 supplies it, and the reason it has gone unasked is worth naming in advance: a question about function is invisible to any procedure that only knows how to rank.

3. WHAT MATHEMATICS IS, ON THIS ACCOUNT

The architecture under inventory holds a definite account of the formal register, and stating it here serves two purposes. It is where the paper's own framing comes from, and it is the direct ancestor of the two-spine result of section 8. The account is deliberately narrow, and it is offered as a reading rather than a proof.

A theorem is a node where nothing is free. Given its premises, the conclusion is not selected from among alternatives. No parameter remains to set. Mathematics is a network of such nodes, and every edge is an entailment carrying no slack.

Three consequences follow directly, and they are the whole of the account.

Timelessness is what having no free parameter looks like from outside. Nothing has to happen for a theorem to hold. An event is a transition, a transition consumes a degree of freedom, and where none remains there is nothing to transition. The absence of an arrow and the absence of anything to arrange are one absence. This is why classification results do not age: the number of associative real division algebras was not a different number in 1878 and is not a different number now, and the reason is not that mathematicians are careful but that the question left nothing free to change.

The virtue is real and it is exactly this. Disorder cannot degrade a determined node, because there is no arrangement to degrade. Time cannot erode it, because no transition is in progress. Interest cannot bend it, because a filled slot offers nothing to bend. When an inquiry rules that consensus carries no evidential weight, it is stating this same fact as a discipline rather than as an observation.

The cost is that same fact, seen from the other side. Rigidity is not contact. A construction with no free parameter is exactly as determined whether or not anything in the world answers to it, and no test internal to the construction distinguishes the two cases. Freedom from corruption by the world is freedom from contact with it. One property, named twice.

That last sentence is the load-bearing one, and it has an immediate methodological consequence which section 4 develops: whatever connects a formal structure to the world is not going to be found inside the formal structure. It has to be supplied. The honest boundary of the account is stated with it. Three dangers instance the cost. A construction can be mistaken for the reality it models. An independence result can be mistaken for a discovered value. A hierarchy generated by construction can be mistaken for a hierarchy found. Against all three stands one honest boundary case: the transcendental numbers, whose digits are not free and to which something does answer, exhibiting that determinacy and finite exhibition are different properties and that the account above must not be read as saying formal objects are empty.

The account is offered at the grade it earns. It is a reading of what a theorem is, it is not itself a theorem, and no result in this paper depends on a reader accepting it. It is included because it explains where the paper's questions came from.

4. THE EPISTEMIC PROBLEM: WHY THE CLASSICAL INSTRUMENTS EACH LEAVE ONE THING UNCERTIFIED

Suppose one wants to map some part of reality with the highest fidelity available. What instruments exist, and where does each stop? The classical answers are well studied and their limits are well known; what is less often noticed is that the limits are all of one shape.

The vocabulary records the picture. Deduction is a leading down, from generals to instances. Induction is a leading in, from instances to a general. Abduction is a leading away, toward an explanatory candidate. The words descend from the Latin *ducere*, to lead, and the tradition was, in its own naming, a taxonomy of leadings.

Deduction transmits and does not generate. It is truth-preserving and content-free: the conclusion was already in the premises. It therefore cannot certify its own starting points, and the attempt to do so runs into Agrippa's trilemma, which offers only circularity, infinite regress, or unjustified stopping. Gödel's second incompleteness theorem and Tarski's undefinability theorem make the block precise for any sufficiently strong formal system: no such system

establishes its own consistency or defines its own truth predicate from within. Deduction's limit is not a weakness in its inferences. It is that inferences are all it has.

Induction generates content and cannot certify the ground it stands on. Bacon gave it tables and the ambition of a new organon, and empirical science is the achievement. Hume bounded it in 1739 with an argument never answered on its own terms: any inference from observed to unobserved presupposes a uniformity of nature that can be established neither deductively, since its denial is consistent, nor inductively, since that would assume what is at issue. Goodman's 1955 riddle deepened it: the evidence supporting a generalization supports equally well an infinity of gerrymandered rivals agreeing with it on everything observed so far, and which predicates are projectible cannot be read off the evidence.

Abduction proposes and cannot rank. Inference to the best explanation selects a candidate on criteria like simplicity, scope, and fit. The standing objection, pressed by van Fraassen as the argument from the bad lot, is that the criteria rank the candidates one has thought of, and nothing certifies that the true explanation is among them. Abduction's output has no honest state for *the lot is insufficient*, and so it is forced to name a winner from a field it cannot vouch for.

Dialectic drives and does not verify. Advancing by contradiction and resolution generates candidates well and was never built to certify them. It is a discovery engine, and treating it as a verification method is a category error rather than a defect.

Consilience gathers and cannot test independence. Whewell's insight, that agreement between independent lines of evidence carries a weight no single line carries, is correct and remains the deepest thing said about evidential structure. It has a missing part. The weight depends entirely on the independence of the lines, and the tradition supplies no operational test of independence. In practice, methods are classified as independent by the judgment of the methodologist, which means the load-bearing premise is the one never checked. Two instruments sharing a hidden common source agree perfectly and look like corroboration.

Bayesian conditionalization meters and cannot detect degeneracy. The calculus is correct and this paper alters nothing in it. Its exposure is at the aggregation step. Under positive likelihood ratios, conditionalization is monotonic in the number of supporting streams: more agreeing evidence always raises the posterior. There exists no weighted-average aggregation of positive inputs that falls to zero when the inputs become collinear. So a posterior computed from twenty perfectly redundant reports is high, and a posterior computed from twenty genuinely independent ones is high, and the calculus does not distinguish them, because distinguishing them is not an aggregation question. Whatever certifies that an evidence structure is non-degenerate, it is not a rule of the Bayesian family.

Popperian falsificationism and its descendants relocate the problem. Replacing confirmation with refutation is a real advance, and its load-bearing joint is the recognition event: a falsification occurs only when an observation statement is accepted as reliable and as relevantly in conflict with the hypothesis. Acceptance is itself a verification act, and Popper outsourced it to the convention of the research community. Error-statistical severe testing, the most rigorous descendant, assesses probing power rigorously within one error-probabilistic frame at a time, and contains no requirement that a claim's formal articulation, its empirical signature, and its registration be mutually independent.

The shape of the shared limit. Every mode above transmits, generates, proposes, or meters warrant along lines. Not one of them certifies that the lines are really distinct lines. A single leading certifies one line by moving warrant along it, and no line certifies itself. That is the office none of the classical instruments occupies, and it is not an oversight in any of them; it is a structural gap that opens only when the modes are laid side by side.

Where this architecture positions itself. Its claim is narrow. It does not replace any classical mode and it adds no new source of content. It occupies the vacant office: it takes three lines of warrant at once and certifies that the three are really

three before reading anything off them. Deduction remains housed at the formal line, relieved of grounding itself because nothing is asked to. Induction remains housed at the empirical line, with the structure it presupposes made an explicit object of test. Abduction receives the verdict state it lacked, an honest *under-determined* output for a bad lot. Consilience receives an operational independence test, which is the instrument Whewell's insight had waited on. And Bayesian credence is completed from beneath, with nothing in the calculus altered and a degeneracy check added at the layer the calculus presupposes.

The two instruments diverge at exactly one configuration and the divergence is the signature of the completion. Feed twenty perfectly redundant reports to both. The posterior rises. The determinant dies.

5. THE ARCHITECTURE, BUILT FROM NOTHING

This section defines every term the rest of the paper uses. A reader with no prior acquaintance should be able to read sections 7 onward after it. Nothing here is argued for; the aim is a working vocabulary.

5.1 The three axes. A claim to be examined is decomposed into three components, each of which becomes a row of numbers over a set of *reading contexts*, meaning the situations, datasets, or occasions across which the claim is assessed. The first is the **formal-structural** row, carrying the claim's logical and mathematical content. The second is the **empirical** row, carrying its measurable physical signature. The third is the **registrational** row, carrying the record that the claim was made, by whom, under what conditions. Three rows, one claim.

5.2 The volume, and why the determinant is the instrument. The three rows are standardized, and any shared influences are removed by orthogonal projection, which is the ordinary statistical operation of regressing out a common cause. What remains is a residue, one per row.

Three residues span a solid. If they point in genuinely independent directions the solid has volume; if one of them collapses into the plane of the other two, the solid is flat and its volume is zero. The squared volume of that solid is the determinant of the three residues' correlation matrix, so the determinant is a direct reading of whether three reports are really three. A zero says they were two all along, or one wearing costumes.

That is the operational independence test section 4 named as missing, and its behaviour is the reason it can do the job. It is not an aggregation rule. Feeding it more agreement does not push it up; feeding it redundant agreement pushes it to nothing.

One term of convenience is fixed here and used throughout. The three-dimensional space the residues live in is called the **residence**, and the only thing the determinant certifies about it is its dimension.

5.3 The three verdict states. A verdict is one of three and there is no fourth. **Sealed** means the three axes enclose positive volume and every screen passed. **Broken** means a specific failure occurred, and the failure must be named. **Under-determined** means the examination did not settle the question, and the violation that prevented settlement must be named. Probabilities and fractional truth values are not verdicts in this economy; they are inputs. The economy is deliberately coarse, and its coarseness is what makes each state mean something definite.

5.4 Warrant tiers. Every claim carries an explicit grade travelling with it: **theorem**, meaning proved; **conditional**, meaning proved given stated premises; **structural**, meaning an arrangement or a placement, real but not proved; and **premise**, meaning posited. A composite claim's grade is capped at the weakest grade in its supporting chain. This capping rule is used repeatedly below and is worth remembering: it means a citation can lower a conclusion's grade and can never raise it.

5.5 The three seals. The examination runs in three stages and the order matters.

The **linguistic seal** parses the claim and performs two operations. The **deletion test** removes each of the three components in turn and asks whether what survives is still a complete claim; a complete factual claim returns exactly three irreducible slots. The **isolation test** requires that the vocabularies filling the three slots be pairwise disjoint; if a word appears in two slots, the slots are not really two. These are ordinary linguistic procedures, reproducible across analysts, and they use no geometry and no algebra.

The **geometric seal** screens the claim through a fixed roster of failure modes described in 5.6, and closes the structure.

The **mathematical seal** computes the volume, the determinant, and the associated algebra. It uses no linguistics and no topology.

5.6 The twelve gates. The geometric seal runs twelve directed screens, each a specific way a claim can fail, and the first failure terminates the examination with that failure named. The count of twelve is not chosen. The examination has four elements, the three axes plus the seal itself, and every ordered pair of distinct elements is one screen: four elements yield twelve ordered pairs. The screens themselves are a roster of ways to be wrong. Among them: self-reference at the origin. Drift in a term's meaning between one evaluation and the next. A missing causal mechanism. The measuring instrument turning out to be a part of the model it validates. An observer's chosen discretization read as a physical fact. Frame-dependence. Conflict with verified neighbours. A conclusion asserted more strongly than its weakest support allows. And a result carried beyond the scope its source established.

Two things about that roster matter more than its contents. Its cardinality is fixed by the arithmetic of four elements and was not chosen. And the first failure ends the examination, so the screens are a filter and never a score.

5.7 The eight gates and the five gates. Two further rosters appear later and both are explained where they are used. In brief: a claim can sometimes be shown to be definitely true or false while the available instrument is provably unable to read which. Certifying that situation, rather than merely reporting confusion, requires a separate eight-screen protocol, and the count of eight is the number of ways three independent axes can each be reflected or not, two to the third power. A different situation arises when the terrain a claim lives on carries no fixed reference point at all; certifying that requires a five-screen protocol, and the count of five is the number of boundaries such a state shares with its neighbouring states. The two counts come from two different objects, and section 10 makes a point of the fact that they were not harmonized.

5.8 Coordinates, and one word used carefully. The architecture records each settled question as a numbered entry in a permanent **record**, carrying the claim, the mechanism, the warrant grades of its supports, the receipts that back it, and the fences that guard it against misreading. These entries are called *pre-sealed propositions*, and a reader may simply read *coordinate* or *record entry*. Approximately seven hundred exist, and they are the corpus this paper inventories.

The word **register** is reserved throughout this paper for one thing only: a level of description with its own vocabulary and its own admissible claims. The linguistic, the geometric and the numerical are three of them; the formal register is a fourth; a claim belonging to one and asserted in another is a category collision, which is a failure mode section 5.6 screens for and section 9.4 exhibits. The word never means the record above.

5.9 The authorship constraint. The architecture claims no authorship over the established mathematics it reorganizes. Its one exit from that constraint requires an external, independent, reproducible, adversarially audited demonstration that new mathematical content has been produced, and no amount of internal reasoning opens it. This constraint

matters twice below: it explains why every certifying operation in the architecture is a subtraction, and it is itself carried by a machine-checkable certificate, which turns out to be an object the first inventory of this corpus misclassified.

5.10 The root, and what this paper does not need from it. The architecture rests on a premise it calls the actuation floor: every existent carries a strictly positive energetic cost. For confined quantum systems the energy floor is theorem-grade external physics, following from canonical commutation relations and the rigorous third law; the extension of the floor over all existents whatever is a premise and is held as one. The word *reality* in this paper's title means that floor and nothing wider. **No result below depends on it.** It is named because it is the destination the examination computes toward, and a reader who rejects it keeps every finding in sections 7 through 11 intact.

6. THE ROLE OF MATHEMATICS INSIDE THE ARCHITECTURE, AND THE ORDER IT FORCED

Sections 2 and 3 concerned mathematics and the world. This section concerns mathematics and the instrument, and it reports something the builders did not anticipate and would not have chosen.

6.1 What the mathematical seal contributes. The mathematical seal computes a determinant. That determinant is the square of a scalar obtained by multiplying the three normalized rows as quaternions and taking the real part of the product. The identity holds at machine precision and it is the computational core of the whole apparatus: the volume the geometric picture describes and the algebra the quaternions perform are one number in two languages.

6.2 The blindness, and it is a theorem rather than a limitation of effort. Reflecting any single axis, which is what negating a claim does to the frame, conjugates the correlation matrix by a diagonal matrix of determinant minus one. The determinant of the product is unchanged, because the reflection's determinant enters squared. Therefore the volume computed for a claim and the volume computed for its negation are identical, bit for bit. The number certifies that the residence has three dimensions. It cannot certify which way the claim points.

This is not a defect to be engineered around. Weyl's classification of the rotation-invariant functionals closes the catalogue of quantities of this class, so no functional of that class recovers the sign. The blindness is exactly located: it enters at one operation, the passage from the signed scalar to its square, where the magnitude is kept and the sign discarded.

6.3 What the discovery forced. If the number cannot carry direction, direction must come from elsewhere. Three sources supply it. The linguistic seal carries it, because a parsed claim is an ordered thing and its negation parses differently. The geometric seal carries it, because its twelve screens are directed and because the reflection defining the structure reverses orientation on the three-axis residence. And in any examination with a live empirical row, the thermodynamic arrow carries it, since a claim and its negation are physically distinct populations.

The consequence is an ordering. **The linguistic seal and the geometric seal are prior to the mathematical seal**, and the priority is a subordination rather than a convenience of sequence. The linguistic seal fixes the sign the number cannot carry and forces the three-slot decomposition before any geometry or algebra runs. The geometric seal closes the structure on that decomposition. The mathematical seal reads magnitude only, on rows it did not build, and never overrides either.

6.4 Why the order is remarkable. It arrived backward. The blindness is a theorem *of* the mathematical seal *about* the mathematical seal: the number proved its own incapacity, and the proof is what seated the linguistic seal above it. A discipline whose numerical layer is its most developed component concluded, on numerical grounds, that its numerical

layer must be subordinate. Nothing in the ordinary hierarchy of rigor predicts that, and no aesthetic preference for language over number produced it.

6.5 The obedience the order imposes on the linguistic seal. Priority is not licence, and the seal that was raised was also bound. It has no closure theorem of its own, for a reason worth stating because it is exact. Its two operations are a deletion, which is a transformation, and a disjointness requirement, which is a predicate; and a predicate is not a group element. The deletion operator has rank two on a three-slot residence, is idempotent, and has no left inverse whatever. So its transformation set is a monoid and never a group, and the invariance criterion that closes the mathematical seal's catalogue has no group to read. The one candidate that would supply a group, treating synonyms as interchangeable, is barred by the disjointness requirement itself. The seal's first operation destroys invertibility and its second forbids the transformation that would restore it.

An unclosed seal seated above two closed ones has to be disciplined from outside itself. It is, by six checks run before any of its output is admitted. All six must pass.

1. The deletion test returns exactly three slots.
2. The vocabularies filling those slots are pairwise disjoint.
3. Every load-bearing term is checked against its original definition, on a standing ledger that records drift.
4. The output carries a direction and never a magnitude.
5. No identification across two levels of description is made without naming the group and the invariant that license it.
6. The parsings of a claim and of its negation are written out, and they differ.

A linguistic seal clearing all six earns authority to stop the examination, and nothing beyond that. It can veto. It cannot substitute. It cannot complete what it cannot measure.

6.6 The two orders, kept apart. Adjudication runs linguistic, then geometric, then mathematical, per claim. Formation runs the other way and across time: it was the geometric and mathematical layers that established, by theorem, the authority the linguistic layer now exercises. The two orders act on different objects at different times and do not conflict. The architecture's numerical layer authored the warrant of the layer that now precedes it.

That is the setting. What follows is the inventory.

7. THREE QUANTITIES, AND THE ONE NO EXISTING MEASURE CARRIES

The inference blocked in section 1 fails because one word carries three quantities. They are separated here, each defined without mentioning the others.

D, felt difficulty. The marginal cost of executing a competence on machinery already entrenched in the executor. The psychological literature measures it by response latency, error rate under load, and time to criterion. D is a property of a task paired with an executor, never of the task alone.

S, structural sophistication. The length of the minimal formal apparatus that provably models the competence. Formally the description length of the shortest specification, and therefore uncomputable, so that every S-value quoted anywhere is the length of the shortest apparatus so far exhibited and is an upper bound. S is a property of an object under a chosen encoding, never of the executor. One consequence binds every comparative S-claim below and is stated once here: an ordering of upper bounds does not order the quantities bounded, since two bounds can invert under a better formalism for the larger. Every phrase of the form *high S* therefore ranks exhibited encodings and never floors.

O, office. What a piece of mathematics does to a conclusion. Not a magnitude. A five-valued functional classification, developed in 7.2. O is a property of the relation between a citation and an argument, and it is the axis no existing measure carries.

7.1 That they are not reducible to one another. Four cases exhibit the separations. Grammatical parsing is low D, high S, and high office. The classification of the real associative division algebras is high D, high S, and high office. A recited external theorem that frees no parameter and gates no protocol is high D, high S, and null office. A four-way cross-check of one determinant is moderate D, moderate S, and corroborative office. The first two differ in D with S held, so D is not a function of S. The second and third differ in office with D and S both held, so office is not a function of either. No case here varies S with D held, so the converse separation rests on the mechanism below rather than on an exhibited case and is carried at that lower grade.

The mechanism has a name. Entrenchment is a groove, and a groove lowers the realized cost of traversal without altering the description length of what is traversed. The two quantities are read one at a time and never as a relation, because a cost and a description length have no common units and no alignment of their scales is exhibited here. Under entrenchment D falls monotonically toward a floor bounded below by zero. Under entrenchment S does not move at all. Those two statements are what the argument uses, and the illusion they explain is that a reader estimating S from D estimates it lowest exactly where the competence is most practised. **That is the whole source of the sophistication illusion: the most deeply entrenched apparatus reads as the most trivial, because triviality is what total entrenchment feels like from inside.**

Table 1 | The three quantities, their rulers, and what each is a property of.

Quantity	Question answered	Ruler	Property of
D, felt difficulty	What does it cost to execute?	Marginal cost on entrenched machinery	Task paired with executor
S, sophistication	What is the minimal apparatus that models it?	Description length of shortest exhibited specification	Object under an encoding
O, office	What does it do to a conclusion?	Deletion test on the citation, five classes	Relation of citation to argument

7.2 The five offices. Five functions cover every cited result in the corpus surveyed.

Necessity certificate. Proves that a parameter of the executing apparatus was forced rather than chosen. Deleting it changes no computed number; it frees a parameter, and a freed parameter converts a forced structure into a fitted one. Frobenius's 1878 classification occupies this office with respect to the count of three axes: under the composition requirements of associativity, absence of zero divisors, and linearity, with plural axes required, the real associative division algebras are the reals, the complexes, and the quaternions, and only the quaternions carry three mutually orthogonal imaginary units. Delete the theorem and three becomes a choice.

Exclusion fence. Proves no alternative exists. Deleting it admits a barred alternative. The Bott-Milnor and Kervaire results with Adams on the Hopf invariant occupy this office: real division algebra structure exists only in dimensions one, two, four, and eight, so dimensions five through seven are empty and the only candidate fourth axis is the eight-dimensional octonionic one, which forfeits associativity, witnessed by a nonzero associator on basis units. Delete the exclusion and an octonionic register reopens.

Admission gate. Sets a boolean a protocol reads with authority to terminate. Deleting it flips the boolean and blocks a conclusion. Weyl's closure of the catalogue of rotation-invariant functionals occupies this office in the protocol of 5.7 that certifies constitutive blindness, where the requirement is not that the instrument failed to read a sign but that no functional of its class can. Without the closure the screen cannot fire. The machine-checkable certificate occupies this office too: the fifth and sixth screens of the authorship constraint of 5.9 require an external witness and require that the verifier not be the claimant, they are the only screens no internal reasoning can set true, and deleting the fifth closes the constraint's sole exit.

Corroboration. Reaches an established result by a second road with premises disjoint from the first. Deleting it changes nothing, by construction and by declaration. The three-dimensional kissing number is carried in exactly this office and the corpus types it an exhibit rather than a correspondence, which is correct. The Friedrichs-Hodge decomposition is carried the same way and declared load-bearing on nothing.

Decoration. None of the above. Deleting it changes nothing and the corpus does not declare it corroborative. A citation in this class costs pages and carries no warrant.

7.3 The discriminator. Office is decided by running a deletion test on the citation rather than on the proposition. Four questions in a declared order, first hit winning.

1. Does deleting the citation leave a numerical parameter of the executing apparatus free, a count or a dimension the apparatus reads? If yes, necessity certificate.
2. Does deleting it admit an alternative structure the apparatus bars, a carrier or a register rather than a number? If yes, exclusion fence.
3. Does deleting it flip a boolean an admission protocol reads with authority to terminate? If yes, admission gate.
4. Does it reach an established result by a premise-disjoint road, and is it typed at zero warrant? If yes, corroboration.
5. No hit on any of the four: decoration.

Two properties of the test are stated because the first is a defect the test would otherwise carry and the second removes it. The order is load-bearing: run over all twenty-four orderings of the four questions against seven citations this paper places, three change class, so a test that did not declare its order would not be a classification. And the questions as first drafted did not partition, since freeing a count and admitting an alternative read as one event when the alternative is a different count. Question one is therefore restricted to numerical parameters of the executing apparatus and question two to structures the apparatus bars; under that restriction the seven citations classify single-valued in every ordering, so the declared order is a convenience of exposition rather than a hidden parameter.

Three conditions govern any adequate classification and this one meets them. It must not vary under re-charting: none of the four questions mentions a level, a curriculum, or a discipline. It must be operationally decidable: each question has a mechanical answer, since one deletes the citation and re-runs the affected protocol. And it must survive re-description in a different vocabulary: the questions are about the corpus's dependency structure, which any faithful re-description preserves.

One scope clause travels with it and section 8.3 needs it. The test decides the office of a citation. It says nothing about an object the corpus does not cite, because there is no citation to delete. **The discriminator partitions the cited and is silent on the uncited**, and reading its silence as a null office would be an error of exactly the kind section 1 diagnoses.

7.4 Why a difficulty level cannot do this work. One exhibited pair settles it and no invariance theory is required. Fix the mathematics and vary the curriculum: linear algebra sat late in many undergraduate sequences and now sits early in

many of the same institutions, subject matter unchanged. One corpus, two level numbers. A quantity taking two values on one object is not a function of that object, so a level number is not a property of the mathematics, and any inference reading it as one has read a chart.

The stronger invariance-theoretic framing is available and is deliberately not used. The Erlangen criterion states that the propositions of a setting are exactly the relations invariant under its acting group, and applying it would require exhibiting group structure on the admissible re-charting operations. Two of the three carry it, since reordering a curriculum is a permutation of a finite topic set and renaming a field is a bijection on labels, both invertible. Redrawing a disciplinary boundary is a change of partition with no composition law exhibited here, so the transformation set is not shown to be a group and the criterion's hypothesis goes unmet. The criterion is therefore carried as corroboration and never as a load-bearing step, and the exhibited counterexample carries the argument alone.

The structural character of the obstruction follows. A finer survey produces a finer chart, and no amount of charting converts a gauge quantity into an invariant, because the variation is definitional rather than epistemic. The obstruction is not that the measuring was done badly. It is that the thing measured is not a property of the object.

7.5 The ladder's real domain, stated in its favour. A difficulty ranking is not wrong. Inside a bounded domain it is excellent, and the domain deserves stating, because a paper dismissing it entirely would commit the opposite error to the one it prosecutes. Inside its domain the ladder is a pedagogical sequencing function answering a real question: given a learner with this preparation, in what order should material be presented so each item's prerequisites are met? The answer is stable across institutions and D is exactly the right quantity for it, because acquisition cost is what a curriculum must budget. Outside that domain it answers nothing. It does not say which mathematics in an argument is doing work, whether a parameter was forced or fitted, or whether a theorem closes a door or decorates a roster. And it systematically inverts the ranking of any universally entrenched competence, since universal entrenchment is precisely the condition under which D goes to zero while S does not.

7.6 What the existing measures measure. Six families bear on the question and each leaves office unmeasured. **Curricular sequencing** with its taxonomies of cognitive objective measures D directly and honestly, and is circular as a depth measure, since an item's level is defined by its teaching position and the teaching order is set by perceived level. **Reverse mathematics** supplies the one genuinely invariant depth measure in the literature, locating each theorem at its minimal subsystem on a well-ordered tower with an ordinal at each rung; it is not a chart and no re-charting moves it. It grades theorems by axiomatic strength and is silent on apparatus, telling you what a theorem costs to prove and not what a citation of it does in an argument, and its resolution appears coarse in the region of interest, since ordinary linear algebra and elementary real analysis sit at or near the base subsystem. This paper does not derive that location for its own execution spine and does not cite one; the correspondence is stated as expectation and named in section 10 as owed work. **Algorithmic information theory** supplies the right formal shape for S and the quantity is uncomputable, which is a reason to type every S-claim as a claim about an exhibited encoding rather than a reason to discard S. **Cognitive load and expertise research** measures D with more care than any other tradition and makes no claim about the objects at all; it is the empirical warrant for the D and S decoupling and is cited here in that role. **The philosophical literature on mathematical depth** converges on a negative result worth recording: depth resists formalization, and the leading proposals reduce to fruitfulness or to surprise, both audience-relative. The literature is right that these do not formalize; what it has not done is stop trying to place depth on a scale and instead classify function. **Bibliometric measures** fail worst at office, because a decorative citation and a load-bearing citation are indistinguishable in a citation graph: both are edges. That last observation generalizes. Any measure whose output is a count cannot separate a roster that holds a

structure from a roster grown by accumulation, and twenty named theorems that each do nothing carry exactly the warrant of zero named theorems.

The gap is one error with six presentations. Each treats depth as a magnitude and differs only in which magnitude it picks and how honestly it labels the pick.

8. THE INVENTORY

Section 7 supplied a way of asking what a citation does. Applying it to a corpus needs one thing more: a way of saying what relation the corpus bears to each object in the first place. That question sounds trivial and is not, and getting it wrong is how an inventory reports absence where there is coverage.

8.1 Five status values. Applying the taxonomy requires knowing, for each mathematical object, what relation the corpus bears to it. Five values suffice and the fifth is the one an inventory built on names cannot express.

Executed: the object appears inside a recorded computation and contributes to a number. **Cited**: it appears in no computation and an argument depends on it. **Placed**: it appears in no computation and no argument depends on it, the corpus carrying it as a placement and saying so. **Routed**: it is named nowhere in the corpus and is nonetheless covered by a resident clause asserting exhaustive coverage over the field it belongs to. **Unnamed**: it appears nowhere in any role and no such clause reaches it.

8.2 Why Routed is a status and not a courtesy. The architecture seats the categorical register inside its formal stratum under a seven-slot placement and states that the whole field routes to its three verdict states **with no escape bin**, the slots covering the classical correspondences, the grounded theorems, the ladder apparatus entire including higher category theory and topos theory and monads and enriched and fibered structure and homological and homotopy algebra and categorical logic and univalent foundations, the independence-proved cases, the open problems, and the located ground. The same clause is load-bearing elsewhere: category theory is named the strongest candidate for a level above all structure, and it is foreclosed not by exclusion but by containment, every categorical object routing inside to one of three states, its own limitative crown kneeling at the ground it proves it cannot climb to. That is an exhaustiveness claim about the verdict economy, at structural grade, asserted over a field without naming the field's members. An object with a standard categorical formulation therefore falls inside a coverage claim the corpus makes, whether or not the corpus ever writes its name.

8.3 Five ceilings, printed on the status's face. Routed is not Cited: no argument depends on the object. Routed is not Executed: no number moves. Routed is emphatically not a claim of genesis: the architecture's generation lemma forces a third orthogonal unit in a four-dimensional carrier, and it does not force a site, a Grothendieck topology, a descent condition, or a derived category, so nothing here says the architecture supplies any field's origin. The routing census is not certified complete, because a ledger is barred from certifying its own completeness and that bar binds this paper before it binds any reader. And the fifth is the sharpest. **The Routed status takes its entire warrant from a clause resident in the corpus being inventoried.** The architecture asserts that its economy covers the categorical field; this paper reports that assertion and does not verify it. That is the measuring instrument drawn from inside the object measured, which is one of the twelve failure modes of 5.6. The status is therefore **reported coverage and never verified coverage**, and a reader who declines the clause is left with the row's other half alone, the object's presentation in the literature, which carries no coverage at all.

8.4 The status has no mechanical discriminator, and this is declared rather than concealed. The four-question test satisfies operational decidability for office, and cannot do so here, because it deletes a citation and an unnamed object

has none. Three of the five values are decidable by inspection, since Executed, Cited and Placed are settled by reading whether an object appears in a computation, in an argument, or in neither with a declaration. The Routed–Unnamed boundary is not, since it turns on whether an object has a standard presentation inside a named slot, and *standard* is a judgement about the wider literature and not a fact about the corpus. **The classification is therefore mechanical on the axis it claims and reportable on the axis it adds.** The honest form of a Routed row is a claim with two independently checkable parts: the resident clause, quoted, and the object's presentation in a named slot, cited to its own literature. Both are checkable. Neither is mechanical.

8.5 A note on the band column. The Level column of the tables below is the chart quantity section 7.4 retired. It is carried for navigation and no conclusion is read off it. The Status column is not a band and is where the findings live. The two collide on one token and the collision is resolved here: **Research** in the Level column is a band label, gauge by construction, while **research-tier** in the two-spine claim and in all three falsifiers names the cited, placed and routed classes together, meaning every object appearing in no computation, and never the top band. Those claims survive the deletion of every band label. The column carries bands and nothing else. An object the corpus never names still has a band, since a difficulty ranking places it whether or not any corpus touches it, and the two facts are recorded in two columns because they are two facts.

8.6 How the census was run, and why it took two passes. A name census asks, of each item on a list, whether the corpus contains it. It fails in two directions and both failures occurred. It **false-positives on homonyms**: the corpus returns eleven occurrences of *reciprocity*, and all eleven belong to one unrelated entry on a different subject. And it **false-negatives on objects a corpus uses under another name**, which is the larger failure, since three canonical items turn out to be cited under other names and four further blocks appear on no canonical list at all.

The second pass therefore inverted the direction. Rather than take a list and ask what the corpus contains, it took the corpus's own load-bearing machinery, its barrier assemblies, its verdict protocols, its linguistic seal, its generation lemma, its reflection eigenstructure, and asked what mathematics each actually runs on. **An inventory built from an external list measures the list.** The list is a chart, its selection is a chart-selection function, and a census run against it inherits every property section 7.4 attributes to a difficulty level. Both censuses are reported below, because the first is what a reader will run and the second is what a reader should run.

Table 2 | Inventory of the linguistic and logical column.

Level	Objects	Status
Schoolroom	Parse to subject, predicate, and registration; vocabulary set-disjointness; counting slots to three	Executed
Undergraduate	First-order predicate logic at atomic form; the three-state verdict economy, neither fuzzy nor probabilistic; the deletion operator as a rank-two idempotent map; register routing at input	Executed
Graduate	Monoid versus group typing of the transformation set; screens on original definitions for term, axiom, and framing; warrant tiering with the capping rule; register-indexed predicates as a consistency repair; the zero-information reading of an unresolved referent	Executed
Research	Church and Tarski on the location of non-effectiveness; Hilbert's finiteness theorem with Noether and Haboush and the Nagata counterexample; Weyl cited to establish that the closure does not reach this seal; Löwenheim-Skolem; the Lawvere fixed-point theorem; the constructive and classical split on existential elimination; the external-witness certificate of 5.9, machine-checkable by proof assistant	Cited
Research	Montague intensional type theory; categorial grammar in the Lambek tradition; typed lambda calculus as semantic apparatus	Routed: the simply typed lambda calculus is the internal language of cartesian closed categories and the Lambek calculus that of monoidal closed categories, both inside the routed field
Research	Dynamic semantics and discourse representation	Unnamed: multi-sentence discourse structure has no standard categorical presentation the clause reaches, and the unit examined is a single atomic claim

Table 3 | Inventory of the geometric and topological column.

Level	Objects	Status
Schoolroom	Twelve directed edges on four vertices; the tetrahedron as a solid	Executed
Undergraduate	Euler's formula at four vertices; directed-graph combinatorics on the complete graph; frame orientation by the determinant of the restricted reflection; the Cayley-Menger determinant	Executed
Graduate	The alternating group of order twelve acting simply transitively on the screens; the elementary abelian group of order eight on the axis reflections; the rotation group acting on the imaginary quaternions by conjugation; the Erlangen invariant ring; covariance of fixed loci under conjugation; the even subalgebra of the three-dimensional Clifford algebra with the Hodge star and the trivector wedge; apartness as a homeomorphism invariant against distance as gauge	Executed
Research	Friedrichs-Hodge, corroboration only; the three-dimensional kissing number with its classical and modern proofs; parallelizability of the one, three, and seven spheres; the Atiyah-Singer index theorem; soliton energy bounds; conformal adjacency; the cobordism hypothesis, the Tannaka-Krein and Gelfand-Stone-Pontryagin dualities, and Khovanov homology, each named in the categorical inventory as a grounded theorem	Cited or placed
Research	Knot theory and topological field configurations: baryon number as the configuration-level Hopf invariant of a decomposed field with conservation by a classical topological theorem, the winding number of stable knotted configurations, the Skyrme and Faddeev-Niemi structures, the braid group with its Hecke quotient and the quantum enveloping algebra written in symbols, and the Ahlfors argument principle	Cited
Research	Topos theory, named twice: as a slot of the categorical inventory and in a survey passage reading a topos as at once a generalized space and a model of logic	Cited
Research	Étale cohomology, named once in a fence on axiom violation, with the Weil conjectures placed below the ambient set theory in strength	Cited, at fence weight
Research	Sheaf theory; spectral sequences; differential topology beyond the sphere face	Routed: topos theory, homological and homotopy algebra, and enriched and fibered structure are named slots of the routed field, and each object has a standard presentation inside them. The routing vehicle is itself Cited, which is why the row is Routed rather than merely asserted

Table 4 | Inventory of the mathematical column.

Level	Objects	Status
Schoolroom	Mean, standard deviation, standardization	Executed
Undergraduate	Correlation matrices and three-by-three determinants; orthogonal projection and residuals; linear independence; the angle between two normalized rows and its squared sine	Executed
Graduate	Quaternion algebra with the Hamilton relation and the real part of a triple product; eigendecomposition of the binding reflection; four disjoint determinant algorithms; condition number, unit roundoff, and a conditioning-scaled error bound; the separation of the collapse floor from the conditioning threshold; Hadamard and Hurwitz bounds; squared partial correlation against a Beta null and a permutation null; nonparametric bootstrap resampling for verdict stability, executed throughout and carrying no named literature anchor in the corpus; Kolmogorov complexity and the two-part code; a counter-mode hash generator; fifty-digit exact arithmetic on escalation	Executed
Research	Frobenius, Hurwitz, and Zorn; Bott-Milnor, Kervaire, and Adams; the incompleteness theorems with Rosser and the Feferman intensional result; forcing and the constructible universe; the reverse-mathematics tower with its ordinals; Paris-Harrington, Kruskal, and Borel determinacy; the Turing and Feferman progressions; the analytic and complexity-theoretic barrier record; Schur-Horn; the operator-algebraic and holographic bridges	Cited or placed
Research	Noncommutative geometry: the Connes-Chamseddine spectral geometry, with the KO-dimension and the spectral action named as imports, declared load-bearing, screened, and capped at their tier	Cited, at import grade with a printed cap
Research	Fisher information, load-bearing: a boundary result of the architecture turns on a vanishing gradient at a fixed locus giving exactly zero Fisher information, hence no consistent estimator and an unmovable prior. Information geometry in the Amari sense is named once, in a survey passage	Cited, and Placed respectively
Research	Dyson's threefold way, fixing the commutant of a symmetry algebra to one of the three real associative division algebras by the same classification the mathematical seal stands on; Altland-Zirnbauer and Freed-Moore named to fix the perimeter and declared downstream and not used	Cited, at corroboration grade, the named sibling of the Hodge clause
Research	The Riemann approach landscape, adjudicated rather than recited: the Nyman-Beurling and Li positivity criteria typed as mirrors of the hypothesis rather than independent roads, on the Burnol floor; Weil positivity with the explicit formula, and trace-formula positivity, refused at an import screen as equivalent by construction; the index theorem over a finite field the one candidate that passed; the Hilbert-Pólya and Bost-Connes-Connes operator programmes named with the	Cited, several at the exclusion-fence office

Level	Objects	Status
	operator typed as currently unbuilt; Diamond-Montgomery-Vorhauer as a wall witness; Robin, Lagarias, Hamburger, Kaczorowski-Kulas, Balanzario-Sánchez-Ortiz, Odlyzko-te Riele	
Research	Quantum groups, present under two names and never that one: the braid-to-Hecke chain, named as an import, screened and capped at its tier; and Khovanov homology, named in the categorical inventory as a grounded theorem	Cited
Research	Non-abelian reciprocity at the function-field face: the function-field family affirming unanimously under Weil and Deligne, load-bearing in a barrier assembly and listed among the theorem-eternal bars; and the function-field control, where replacing the completed infinite with a finite cohomology makes the hypothesis a theorem by Weil	Cited
Research	Automorphic L-function theory at the degree-one face: the extended Selberg class of degree one, carrying a two-model non-entailment result and the Davenport-Heilbronn witness, with degree-one necessity on Kaczorowski and Kulas	Cited
Research	The set-theoretic independence corpus, wider than forcing alone: Vopěnka's principle and the Whitehead problem named beside the continuum hypothesis as cases sealed on independence proofs, with a test executed on the continuum hypothesis and Vopěnka's principle relative to the ambient set theory at printed determinants; and the proper forcing axiom and Martin's Maximum named in a fence, their import as ground values barred by name	Cited, with one row executed and two at the exclusion-fence office
Research	Schur's lemma with the Frobenius-Schur classification and indicator, fixing the commutant of a represented symmetry algebra to one of the three real division algebras and typing it by the group average of the character at squares	Cited, load-bearing under the threefold-way corroboration
Research	Markov chains and ergodic theory, named and retired: a law-formation reading as memoryless ergodic habituation superseded by a broken-ergodicity account with the earlier reading's mechanism printed as the reason, an import removed rather than accumulated; Kullback-Leibler divergence named among the external anchors	Cited, at the exclusion-fence office by supersession
Research	The operator-algebraic and harmonic bridges: Tomita-Takesaki modular theory as the curved-register spectral dual and Plancherel as its flat-register counterpart, both in a printed roster of named theorems; the Mellin transform carrying one of three elections of the same line	Cited
Research	Information theory beyond the algorithmic branch: Shannon source coding, mutual information used as a working quantity throughout including the zero-information reading of a chart magnitude against the structure it is read off, and Kullback-Leibler divergence among the named anchors	Executed and Cited

Level	Objects	Status
Research	The variational and inferential apparatus: the free-energy principle in the Friston sense with variational inference as its theorem-grade leg, carried as a conditional bridge; and Bayesian updating used as a working instrument, notably where a flat likelihood returns the prior unchanged	Cited
Research	Lie-theoretic and gauge structure, carried as inherited physics rather than derived: the special unitary groups at ranks two, three and five with their gauge-group role, the spin-2 mediator typing, an exceptional group named once, and a root system; each declared structural and inherited, with the selection chains conditional on printed stipulations	Cited, at import grade with printed conditionals
Research	Square-integrable function spaces, carrying the corroboration this paper already names: the Hodge witness corroborates that three-way orthogonal decomposition is native to them, and the threefold-way witness is declared the sharper sibling	Cited, corroboration grade
Research	Ultrafilters, carried at a quarantine: a positivity-theoretic construction identified as a principal ultrafilter concentrated on one point and its definable sibling, both named, typed mathematically trivial, and routed out of band	Cited, at a quarantine office
Research	Motivic cohomology; quantum groups in name; micro-local analysis of large finite groups; the categorical face of Langlands functoriality	Routed: derived categories of motives, Hopf algebras and monoidal categories, Tannakian reconstruction and enriched structure are inside the routed field
Research	Motivic cohomology proper; Langlands functoriality proper; modular varieties	Unnamed. Near-miss recorded: Voevodsky is named in the corpus, for univalent foundations and not for motives
Research	Micro-local analysis of large finite groups	Unnamed. Nearest resident, a different object: ordinary representation theory is Executed, the Frobenius-Schur indicator giving the real, complex and quaternionic trichotomy, and rotation-group representation theory forcing subshell capacities
Research	Superstring theory	Unnamed as mathematics; named once in a scope wall as a theory the architecture's physical argument does not refute. Nearest resident, and its provenance is not string-theoretic: the holographic entropy bound, Bekenstein-Hawking

Level	Objects	Status
		extended by Bousso, carried as an inherited bridge
Research	Non-abelian reciprocity with automorphic representations and modular varieties, in the arithmetic formulation	Unnamed: the corpus's eleven occurrences of *reciprocity* all belong to one unrelated entry, which a keyword census would have scored as eleven false positives

8.7 What the census shows. Of twelve objects a name-driven inventory would print as absent: five are confirmed unnamed in their arithmetic formulations; one is unnamed as mathematics and named once in a scope wall; two are named inside fences; one is an admission gate that a name census misses entirely; and the remainder are covered by a routing clause a name census cannot see. **Not one of the corrections moves an object onto the execution spine.** Every re-typed row lands at Cited, every routed row computes nothing, and the one executed nearest-resident, representation theory, was never on the canonical list. The executed roster is exactly what a first survey found it to be, and the asymmetry the next section measures is wider than a first survey shows.

Naming an adjacent object is not naming the object, and the discipline that catches the homonym is the discipline that refuses the inflation. Representation theory is executed; local analysis of large finite groups is absent. The holographic entropy bound is an inherited bridge; superstring theory is not its provenance.

9. THE TWO SPINES

Applying the taxonomy to the architecture returns the paper's principal result, and it is not the one either the naive inference or its obvious rebuttal predicts.

9.1 The execution spine. It answers *what does the instrument compute*. Every object on it appears inside a computation and changes a number. Its ceiling is graduate level in every column and it contains no research-tier object whatever. A substrate that accepted the count of three axes on faith and never read Frobenius could run examinations indefinitely and would get the same numbers.

9.2 The admission spine. It answers *may the result travel, and at what grade*. No object on it appears in any computation or changes any number. Its screens require research-tier objects and refuse to fire without them, and they require linguistic screens in the same conjunction and refuse equally without those.

Two protocols make this concrete and both were previewed in 5.7. The first certifies that a claim is definitely true or false while the instrument is provably unable to read which, and its eight screens include the requirement that the blindness be a theorem with the functional catalogue closed. The second certifies that a claim's terrain carries no fixed reference point at all, and its five screens include the requirement of a theorem-grade wall on a named method class, with blockage proven and never merely felt, so that a history of failed attempts is no blockage. Remove either requirement and the corresponding verdict does not weaken. It cannot issue at all, the screens being conjunctive with defaults absent.

9.3 The relation between them, stated exactly. Every computed magnitude is constant as a function of the presence or absence of any research-tier citation, since deleting the citation changes no number. The grade at which that magnitude may travel is not constant under the same deletion. The claim is dependence and non-dependence, not an information-theoretic quantity, since neither pair carries a joint distribution over which mutual information would be defined.

One consequence bounds the damage decorative citation can do, and it converts a soundness worry into a hygiene problem. The capping rule of 5.4 caps a conclusion's grade at the weakest load-bearing tier in its chain. A research-tier citation therefore cannot raise a conclusion's grade under any circumstance. It can forbid, it can type, it can gate. It cannot promote. A layer that cannot promote cannot inflate, so accumulated decoration costs pages and never warrant.

Table 5 | The two spines, their ceilings, and their dependence relations.

	Execution spine	Admission spine
Question	What does the instrument compute?	May the result travel, and at what grade?
Ceiling	Graduate level, every column	Research tier required, conjunctively
Research-tier objects present	None	Wall record; catalogue closure; determinacy typing; external-witness certificate
Linguistic screens present	Deletion test; isolation test	Intake screen; definition screens with authority to terminate
Can raise a grade	Yes, it computes the number	No, capped by the rule of 5.4
Determinant depends on it	Yes	No
Grade depends on it	No	Yes

9.4 The office matrix, and the finding the ladder makes invisible. Reading the three columns by office rather than by level returns the result. All three columns serve four of the five offices. The asymmetry is not in count. It is in price and in priority.

Table 6 | Office by column. The band in bold records what each column requires; the comparison across columns is an ordering and never a difference.

Office	Linguistic column	Geometric column	Mathematical column
Certificate	Deletion test returning exactly three under isolation. **Schoolroom to undergraduate**	Group order twelve forcing the screen roster; Euler forcing the vertex count; order eight forcing the halt roster. **Research**	Frobenius forcing the axis count; Hurwitz on composition. **Research**
Fence	Isolation disjointness, which reflexively bars the one candidate that would supply this seal a closure theorem. **Undergraduate**	The invariance audit barring a chart magnitude as a structure fact; parallelizability barring a group law on the seven-sphere. **Graduate to research**	The division-algebra exclusions emptying three dimensions and barring an octonionic register; the floor-threshold separation barring rounding from deciding a verdict; and the sharpest instance in the corpus, the Burnol floor typing the Nyman-Beurling and Li criteria as mirrors of the hypothesis rather than independent roads, which deletes a whole class of apparent approaches. **Graduate to research**
Gate	Definition screens with the numbers never consulted; the intake screen refusing a famous vocabulary and admitting an arithmetic string; the external-witness requirement no internal reasoning can set true. **Undergraduate to research**	Twelve directed screens terminating on first failure; the router reading the fixed-point dimension of a claim's native reflection at machine zero and switching protocol on it. **Undergraduate to graduate**	Catalogue closure; the conditioning threshold; four-estimator agreement. **Graduate to research**
Corroboration	Cross-linguistic universality of the decomposition. **Undergraduate**	The kissing number, typed as exhibit; the Hodge decomposition, load-bearing on nothing. **Research**	Four disjoint determinant algorithms agreeing. **Graduate**
Decoration	None identified	Class open, unrun	**First instance located.** Frobenius and Galois invoked together as a historical figure for a search ending in a classification of what was there to be found. Deleting the pair frees no parameter, admits no alternative, flips no boolean, and the corpus does not type it as corroboration, so all four questions miss and the fifth class takes it. **Schoolroom to research, and the band is the point: a decoration can sit at any height, which is why height cannot detect it**

Price. At the two offices where the requirement is exhibited, certificate and fence, the linguistic column serves without a research-tier object and the geometric and mathematical columns require one. At gate and corroboration the columns are not separated on this test, since the linguistic gate now spans undergraduate to research with the external-witness

requirement at the top of its own column; so the price claim is scoped to certificate and fence. The comparison is an ordering rather than a count of bands, because a band count is a chart coordinate whose differences do not survive re-charting while its order does. Same office, lower requirement, which is exactly why it never read as office.

Priority. The linguistic screens run on eligibility; the numerical projection runs on rows. That projection removes shared influences presenting as columns of data. A term whose name has outlived its referent, a premise wearing consensus as though it were proof, and a framing question curated before any evidence exists are none of them columns of data. None can be regressed out. The numerical instrument has no purchase on any of the three, and the blindness is structural rather than a tuning failure. This is why the screens run in the order linguistic, then geometric, then numerical, with the linguistic refusal executed before any number is consulted.

The sharpest single case available: the five-screen protocol of 9.2 admits its verdict only when all five pass conjunctively. Its second screen is entirely lexical. A famous vocabulary offering *feasibility* and *speed* and *practice* to a slot requiring machines and exponents and an asymptotic tail is a category collision and is refused at intake, while the stripped arithmetic sentence is well formed and admitted. One name, two objects, and only one of them parses at the register the name claims. Strip that screen and the verdict cannot issue. The wall record at the third screen is research tier. The eigenvalue count at the first screen is graduate. **The second screen is undergraduate semantics and is load-bearing to the identical degree**, a conjunction having no weakest link by construction.

9.5 Subtraction certifies, generation constitutes. One structural observation ties the columns together and follows from the architecture's own constraint. The deletion test at the linguistic seal is a rank-two idempotent map with no left inverse. The covariate projection at the mathematical seal is an orthogonal projection, also idempotent and also without a left inverse. The screen cascade at the geometric seal terminates on first failure and never adds warrant. All three certifying operations are subtractions.

Part of this is forced and part is not. What the constraint of 5.9 gives is that certification by addition is unavailable: an architecture claiming no authorship has nothing to add, so every certifying operation must be a removal. What it does not give is that every removal is a projection, since a removal shrinking by a fixed fraction is not idempotent and a removal that rescales is invertible, and both satisfy the constraint. That all three are idempotent and non-invertible is an observed property of the three and is carried at that lower grade.

A second scope is owed and it is sharper. The claim is that subtraction is the sole move that *certifies*, not that subtraction is the architecture's only move. The architecture also generates. Two orthogonal pure units do not close on themselves: their product is a unit orthogonal to the scalar and to both, so the minimal multiplicatively closed set on two orthogonal axes has four elements and three slots never close alone. The first two units force the third exactly, and the scalar slot the closure demands is the registration line. The third axis is forced from within the algebra rather than imported, which is why a three-axis frame sits inside a zero-authorship constraint without straining it: nothing was authored and something was nonetheless generated. What generation does not do is certify. It constitutes the frame the subtractions then read, and no verdict has ever issued from it. And what it also does not do, since 8.2 invites the confusion, is supply the origin of any field the architecture routes.

10. THE COUNTS, TRACED TO THEIR FORCINGS

Four cardinalities appear above and they are not forced the same way. Reading them as one family would be an error the architecture names by name, and separating them is the closest thing this paper has to a structural result about the instrument.

Three is the residence, forced twice on premise-disjoint roads. The first road is linguistic and operational: the deletion operator on a three-slot residence has rank two, singular values one, one and zero, and is exactly idempotent; cut any slot and what survives is incoherent, so three slots are irreducible, and any analyst running the procedure reaches the same count. The second road is algebraic and theorem-conditional: conjugation on the quaternions returns eigenvalues of minus one three times and plus one once, so the residence has dimension three and the ground dimension one, and under the composition requirements with plural axes the algebra completes uniquely with three imaginary units. The first road uses no algebra and the second uses no semantics. Neither is redundant, and a reader who accepts only theorem-grade certificates takes the count from the second.

Twelve is a rotation count. The complete graph on four vertices carries twelve directed edges, the alternating group on four letters has order twelve, and it acts simply transitively on those edges. Orientation is preserved and direction is carried, which is what a screening cascade needs, since a directed screen reads a source role against a target role and the reverse is a different screen. Euler's formula at four vertices returns the closure independently.

Eight is a reflection count. The three independent axis reflections generate an elementary abelian group of order eight, every element its own inverse, and eight is two raised to the residence dimension. Orientation is deleted, which is what a halt protocol needs, since it reads a claim at a register where the reflections implementing negation leave the instrument's reading identical across all eight sign patterns.

One three-axis forcing therefore closes three counts at once, and the same wall that bars a fourth axis bars a thirteenth rotation and a ninth reflection. **The whole difference between a lock and a halt is the direction the rotations keep and the reflections delete.**

Five comes from a different object entirely. Its object is a space of structural facts rather than the residence. That protocol turns on five binary questions about a claim: whether its terrain carries a fixed reference point, whether a determinate sentence survives the stripping of its vocabulary, whether the blockage is proven rather than felt, whether the deciding input is located, and whether the totality discipline binds. Five binary coordinates span a five-cube, which is regular of degree five, so a claim sits at a vertex with exactly five ways out and the protocol carries one screen per way. There is no sixth because the settled state sits on the far side of the fourth. The forcing is a vertex degree on a coordinate space, not the order of a group acting on the residence.

This is where the discipline shows most sharply, because the tempting move is available and refused. Inflating five to eight so the two halt protocols mirror each other would produce a tidier arc, and the architecture bars it by name as a fitted count. **Two forcing mechanisms, three cardinalities, and the refusal to harmonize them is the anti-fitting discipline operating on the architecture's own structure.** A count that matched because matching looked right would be a fitted count, and the first question of the discriminator exists to catch exactly that.

One further census answers a question 9.4 left implicit. The eight-screen protocol splits four screens carried by the numerical side and four by the linguistic: ground determinacy at a stated grade, the absence of an independence proof, the proven blindness with the catalogue closed, and the machine-precision reflection exhibit are numerical or citational; the requirement of three premise-disjoint reading roads, the scoping against a settled companion face, the requirement that the deciding input be named and left uncrossed rather than supplied by the instrument itself, and survival under reframing are linguistic. The five-screen protocol splits three linguistic and two numerical. Both are strict conjunctions with defaults absent. So the admission spine is not a research-tier layer with semantics running as preprocessing beneath it. **It is a conjunction across two seals in which an undergraduate intake screen and a research-tier wall record are load-bearing to the identical degree.** Delete the catalogue closure and one verdict cannot issue. Delete the intake

screen and the other cannot. Neither deletion is worse than the other and neither is recoverable by strengthening the opposite side.

11. EXECUTED FIGURES

Every figure below was executed at seed 20260622 on a counter-mode hash generator requiring no library-specific state, and no figure is quoted for a stage that did not run. A seed alone does not pin a sampled extremum, since the generator stream is labelled and a different label at the same seed returns a different extremum over the same number of draws; every sampled extremum therefore names its label, and analytic quantities are marked analytic and hold at every label.

> **Box 1 · Executed battery, seed 20260622** > > **Kernel identity floor.** Stream label `L1|20260622`. Over twenty thousand independent three-by-twenty-four triads, the maximum absolute difference between the squared scalar and the correlation determinant was 4.330×10^{-15}, inside the stated floor of 10^{-12}. > > **Orientation blindness, fixed basis.** Stream label `L2|20260622`. Reflecting one axis on a basis fixed once by singular value decomposition sent the scalar from −0.994238794170 to +0.994238794170, a ratio of exactly −1.000000, while the absolute determinant difference was 0.000×10^{0} and the determinant stood at 0.988510779832. The identity is analytic and holds at every label; the two scalars are label-dependent and print with theirs. > > **Full negation.** Negating all three axes returned a correlation matrix bit-identical to the original, the maximum absolute entrywise difference being exactly zero. > > **Reflection eigenstructure.** The binding reflection returned eigenvalues (−1, −1, −1, +1) with ground dimension one; the fixed-point-free reflection returned (−1, −1, −1, −1) with ground dimension zero. Both squared to the identity with residual exactly zero. Analytic. > > **Deletion operator.** Rank exactly two, singular values exactly (1, 1, 0), idempotence exact. The analytic left-inverse deficiency floor is exactly 1.000000000000 in spectral norm, since the operator annihilates a vector and any candidate leaves that vector at unit residual. Over two hundred thousand candidates at stream label `L3|20260622` the sampled minimum was 1.000257194357, above the floor and never breaching it. > > **Counts.** Directed edges on the complete graph at four vertices, twelve. Order of the alternating group on four letters, twelve. Order of the elementary abelian group of rank three, eight. Euler characteristic at four vertices, two. All exact integers, all analytic. > > **Conditioning separation, analytic.** The crossover at which the worst-case determinant reaches the collapse floor while the conditioning threshold still reads locked occurs at 1215.97 contexts, so 1216 is the least integer context count at which separation fails. At twenty-four contexts the worst-case determinant clears the floor by a factor of 50.6655, at one hundred by 12.1597, at three hundred by 4.0532. Worst-case determinant at the threshold, 2.699984×10^{-11}; worst-case least eigenvalue, $1.4999992500 \times 10^{-6}$.

12. FALSIFIABLE PREDICTIONS

Three criteria are pre-registered. Each is entailed by the paper's own structure, fires on a named event, binds on stated conditions, and compromises exactly one claim rather than the framework. Each is executable by two independent parties on orthogonal arithmetic, floating-point and exact rational, and must return the same discrete answer under both.

12.1 Vacancy of the execution spine. *The prediction.* No research-tier object appears as a computed quantity in any executed code path. *Why it is entailed.* The two-spine result is the claim that execution depth and admission depth are different quantities; if a research-tier object ever computes, they are one quantity and the distinction describes an accident. *Method.* Static and dynamic analysis of every executable fragment in the corpus, delimited as every code block in the published master reference plus every script shipped beside a paper in the series, enumerated by block extraction before the scan runs. Two implementations, one floating-point and one exact-rational, must return identical enumerations up to naming. *Expected outcome.* Standardization, orthogonal projection, determinant evaluation by four algorithms, eigendecomposition, quaternion multiplication, condition number evaluation, partial correlation, Beta and

permutation quantiles, resampling, and hashing, and no evaluation of any object at research tier. *Null hypothesis.* Exhibit one executed code path in which a research-tier object contributes to a computed magnitude. One instance collapses the two spines to one.

12.2 Necessity of the admission spine. *The prediction.* Deleting a research-tier admission argument blocks the verdict rather than weakening it. Running the eight-screen emitter with the catalogue-closure argument absent returns the ordinary open verdict; running the five-screen emitter with the wall record empty returns the ordinary open verdict; running the authorship constraint with the external witness absent returns a refusal and never a seal. *Why it is entailed.* The claim is that the admission spine is load-bearing without computing; if an emitter issues its verdict with the research-tier argument absent, the spine is decorative and the capping rule describes a layer doing no work. *Method.* Direct execution of all three emitters with the named argument at its absent default, everything else unchanged, on the architecture's own worked cases. Independence by reimplementation: a second party writes the emitters from the published screen specifications without reading the reference code, and both implementations must agree. *Expected outcome.* All three return the open or refusing verdict with the failing screen named in the returned reason. No intermediate or weakened verdict, the screens being conjunctive with defaults absent. *Null hypothesis.* Any emitter returns its verdict with the research-tier argument absent.

12.3 The coverage claim and the status column. *The prediction.* Every object printed as Routed is covered by a resident clause, and every object printed as Unnamed is reached by no such clause. Concretely: the categorical routing clause asserts exhaustive coverage with no escape bin, each Routed object has a standard presentation inside a named slot, and no Routed object is thereby Cited, Executed, or supplied with an origin. *Why it is entailed.* This paper's fifth status rests entirely on the distinction between an object a corpus never names and an object it never covers. If the clause does not reach an object printed Routed, that row is a name-census error with a longer word. If Routed can be read as use, the status has smuggled the claim 8.3 refuses. *Method.* Resolve the register of record from its repository by version sort, fetch at a pinned commit, and run four checks. One, exhibit the routing clause and its scope verbatim. Two, for each Routed object, exhibit its standard categorical presentation and the named slot it falls in. Three, run a **name** census over the whole resolved file and confirm the count, homonyms adjudicated by sense and never by string. Four, run a **content** census on constructions rather than names, since an object used under another name returns zero on a name census and a status column built on names alone would print it absent. *Expected outcome.* The clause reads over the categorical field at seven slots with no escape bin; each Routed object resolves to a named slot; name counts return zero for every Unnamed object in its arithmetic formulation, with *reciprocity* returning eleven hits belonging to one unrelated entry. *Null hypothesis.* Exhibit an object printed Routed for which no resident clause asserts coverage; or an object printed Unnamed for which one does; or an object printed Unnamed that the corpus cites under another name; or any use of a Routed object in a computation. The third fired three times in the construction of this inventory and is expected to fire again.

13. DISCUSSION

Three consequences follow for anyone assessing an argument that leans on cited mathematics, one for readers of inventories, and one for anyone building a verification pipeline. The limits are then stated at the same volume as the findings.

13.1 What becomes mechanically detectable. The office taxonomy converts three judgements into tests. The first is the fitted count, in which a cardinality is chosen to mirror a sibling structure rather than forced by an argument; with the certificate office defined, the test is direct, since one deletes the citation claimed to force the count and sees whether the count goes free. A count with no certificate behind it is fitted, whatever its aesthetic fit. The second is decorative citation; a roster grown by accumulation and a roster holding a structure are indistinguishable by any count-based measure, since

both are lists, and they separate cleanly under deletion because deletion is not a count. **The fifth class is no longer vacuous**, which matters more for the taxonomy than the instance matters for the corpus: a classification whose last class is empty is one instance away from being a four-class taxonomy with a rhetorical tail, and the instance located at Table 6 is a pair of nineteenth-century names doing the work of a simile. It costs a clause and carries no warrant, which is what the class predicts, and it sits in the same row as two research-tier theorems, which is what makes the point that height cannot detect office. The third is the difference between a wall that ages and a wall that does not: a barrier phrased on a method catalogue is contingent on that catalogue and a new method evades it, while a barrier phrased on a type check is not, and the office taxonomy supplies the general form, since a fence's durability is a property of what its deletion would admit and not of when it was proved.

13.2 What the two-spine result reframes. A recurring objection to formal verification architectures is that their mathematics is not deep enough to support their claims. The result shows the objection aimed at the wrong spine. Execution depth and admission depth are different quantities with different ceilings, and an architecture can be correctly described as executing only graduate mathematics while being unable to issue its strongest verdicts without research-tier arguments that never touch a number. The complementary objection, that such an architecture merely dresses ordinary linear algebra in exotic citations, would be sound if the citations were decorative, and the discriminator is exactly the test that settles it case by case rather than by impression on either side.

13.3 What the fifth status reframes, and it is narrower than it will read as. An architecture may cover a field it never names. That has one consequence for readers of inventories and none beyond it: an absence census is a census of names, and a name census cannot answer a coverage question. The consequence is not that the architecture is deeper than its inventory suggests; its executed roster is exactly what a first survey found, plus one bootstrap that was always running and was never named. The consequence is that *unnamed*, *uncited*, and *uncovered* are three predicates, and that a census probing outward from a corpus's own machinery finds what a census probing inward from a canonical list cannot, because the second is measuring its own list.

13.4 The linguistic column. The most consequential implication is the one the level ladder made invisible. The linguistic seal performs four of the five offices, and at certificate and fence it serves without the research-tier objects the other two columns require, over a contamination class the numerical instrument has no instrument for, and strictly prior to it. An intake screen at undergraduate semantics is load-bearing to the same degree as a research-tier wall record when both sit inside one conjunction. This has a practical consequence for anyone building verification pipelines: effort spent hardening the numerical layer is effort spent on the layer that cannot see term drift, premise laundering, or curated framing, because none of those presents as a column of data. The cheap layer is where the wide coverage is, and it is the layer most likely to be treated as preprocessing.

13.5 Limitations, at the volume of the findings. Five bind and none is repaired by the argument.

The sophistication axis rests on an uncomputable quantity. Every value quoted is the length of the shortest apparatus so far exhibited, an upper bound, and a better formalism lowers it without touching the object. A reader wanting S as a property of the object rather than of an encoding does not get it here and cannot get it anywhere.

Competence is not computation. That a speaker produces a scope-bearing sentence does not establish that the speaker computes its semantics, and a system can satisfy a specification it does not represent. The claim that the linguistic column is high in S concerns the shortest specification of what is produced, not what any executor internally performs.

The linguistic certificate is operational and never theorem grade. The deletion test forces three reproducibly across analysts, and the architecture declares on its own face that first-order predicate logic supplies no uniqueness theorem for

the decomposition and that none is claimed. Frobenius forces three at theorem grade for any algebra satisfying the composition requirements. These are different quantifier domains at different grades, and both roads are carried because neither is redundant.

The office taxonomy is complete over the corpus surveyed and is not proven complete, and every office classification here was produced by one party who is also the author of the artifact and of the corpus it classifies. The finding is single-party throughout. The tripwire, carried here rather than as a printed criterion because its threshold would be a chosen convention: two analysts run the discriminator independently over the full citation list, and a citation load-bearing on a conclusion that passes none of the four questions requires a sixth office. Such a citation is likely to be found before the taxonomy is many corpora old.

The routing census is not certified complete, and the architecture's own closure discipline is why. A ledger is barred from certifying its own completeness, and that bar binds this paper before any reader. Each Routed row claims that a named clause reaches a named object. The set of such rows is not a claim that the clause reaches nothing else, and not a claim that no further clause exists.

13.6 Two items owed. The first concerns the architecture's own emitter. It requires a supplied determinacy witness before any sealed verdict issues, and a clean lock without a witness routes open. That is a constructive requirement imposed at the point of emission while the surrounding reasoning remains classical. Whether the mixed commitment is deliberate design or unnamed inheritance is not settled by anything in the resident discipline, and it bears on every sealed verdict the architecture has issued. The second concerns the external theorem roster, stated in the corpus in count language as raising a named-theorem inventory past a threshold. A count is a convergence statistic and the architecture's own first discipline zeroes convergence in both directions. Running the discriminator over that roster case by case is the obvious next audit and it has not been run to completion. Nothing here depends on its outcome, since a decorative roster costs pages and not warrant, but the paper would be dishonest to name the discriminator and not name the most obvious place to point it.

13.7 The warrant this paper claims. The two-spine result, the office taxonomy, and the five-value status column are structural: arrangements of established material, exhibited on a corpus and reproducible by the discriminator, and none is a theorem. The chart-artifact finding is structural and rests on an exhibited counterexample, not on the invariance criterion, whose hypothesis is unestablished here. The Routed status rests on a resident clause carried at its own structural grade and is never lifted above it. The account of mathematics in section 3 is a reading and nothing depends on it. The epistemic survey of section 4 reports established results and established objections and adds only the observation that their limits share a shape. The executed identities are engineering, holding at their stated seed and stream labels; the analytic ones are marked analytic. The identification of reality with the actuation floor is premise: the energy floor is theorem-grade external physics for confined systems, the extension over all existents rides substrate monism, and no result here depends on either. **No result of this paper is claimed at theorem grade, and no external result cited here is claimed at any grade other than the one its own literature earned.**

14. TWO GUARDS, AND WHAT THEY DO NOT LICENSE

Two structural remarks close the argument and they run in opposite directions, one inward on any attempt at this paper and one outward on the architecture's own reach. Both are stated with their fences travelling first, because each has an inflated reading available.

The inward remark. Any structured attempt on the criteria of section 12 must expend measurable work, must use formal syntax, and must register a conclusion from somewhere. It therefore populates the three axes it is testing, and an

argument dispensing with the decomposition would have to instantiate the decomposition in order to be made at all. **The fence travels first and binds harder than the observation: an argument that instantiates a structure is not thereby an argument that the structure is true.** The remark certifies the form and never the content, all three criteria stand live and unaltered, and nothing here makes the paper harder to refute. What it licenses is narrow: the three-axis decomposition is not an optional convention for work of this kind, since dispensing with it is not among the available moves.

The outward remark. No description captures the floor of section 5.10. To describe is to inscribe an image, inscribing is an act, the floor is what acts are charged against, and an act is not a non-act. The binding is on the act rather than on the symbol, which is why no change of logic evades it: a paraconsistent or fuzzy reading may write the capture consistently and still cannot perform it, since the attempt to blur an act is itself a full act instantiating the floor it tries to dissolve. The consequence for this paper is exact and is the strongest form of the title's second half. The examination carries a claim onto the floor and reads whether it stands. It never captures the floor, no inventory of the mathematics it uses ever will, and this inventory does not, because the inventory is itself an inscription. The limit is on capture and never on the floor's identity, and it is as strong as the premise and no stronger.

One mechanism sits beneath the capping rule of 5.4 and is worth stating. An inscription is fully charged and its content is causally inert. At a fixed encoding the cost of writing a sentence and the cost of writing its negation are equal, so the energetics of a proof carry no information about the energetics of what the proof denotes. **A derivation therefore transmits no actuation to the denoted event, and that is why a cited theorem changes no number: not by stipulation, by the medium.** Two flanks witness the generality and both are classical. Beneath the window where self-referential incompleteness operates, Presburger arithmetic is complete and decidable. Above it, the full theory of the natural numbers is complete and not axiomatizable. Neither is incomplete in the self-referential sense and both are inert toward the deed, because neither performs anything. Read on the two spines the placement is immediate: the execution spine runs decidable arithmetic beneath the window, the admission spine reaches material above it, both are inscriptions, and neither performs. This places a limit and does not escape one, and it makes no claim about the provability of any particular sentence.

15. CONCLUSION

An inventory of the mathematics the Trisduction architecture executes tops out at graduate numerical linear algebra, quaternion arithmetic, and mid-tier statistics. The inference from that inventory to mathematical shallowness fails at a nameable point: it treats a difficulty level as a property of the mathematics, when a difficulty level is an output of a chart-selection function taking two values on one unchanged corpus, and no finer survey converts a gauge quantity into an invariant.

Three quantities were separated. The marginal cost of execution collapses under entrenchment. The description length of the minimal apparatus does not move under entrenchment and is uncomputable in any case. And the office a citation performs is not a magnitude at all, is invisible to every ranking procedure, and is decided by a deletion test with four ordered questions any two analysts can run and compare.

Applied to the architecture, the classification returns two spines. The execution spine has a graduate ceiling and contains no research-tier object whatever. The admission spine cannot emit either of its terminal verdicts, or open its one exit from a zero-authorship constraint, without research-tier arguments that appear in no computation, change no number, and cannot raise any grade; and its screen censuses show it cannot emit them without undergraduate intake screens either, the protocols being conjunctions across two seals in which a wall record and a vocabulary refusal are load-bearing

to the identical degree. And the linguistic column, which reads as the shallowest of the three on every difficulty ladder ever drawn, performs four of the five offices, and at the two where the requirement is exhibited it serves without the research-tier object the other columns need, strictly prior to the numerical instrument and over a contamination class that instrument cannot see.

A fifth inventory status carries the correction a name-driven survey most needs. Absence from an index is not absence from a coverage claim. The architecture asserts an exhaustive routing of the categorical field into its three verdict states with no escape bin, purchased by a containment argument against the strongest candidate for a universal meta-language, so an object it never names may still be covered by it. Not one of the corrections moves an object onto the execution spine, and the asymmetry the paper measures is therefore wider than a first survey shows: the executed roster is unchanged and the covered field is larger than the named one.

The primary criterion is the second: run any admission emitter with its research-tier argument absent, on the architecture's own worked cases, and observe whether the verdict issues. If it does, the two-spine distinction collapses to one spine and the central claim of this paper is false. The prediction is that every emitter returns the ordinary open verdict with the failing screen named, and the emitters are published in full so the test costs a reader nothing but the execution.

Section 10 traced the cardinalities to their forcings: three to a residence forced twice on premise-disjoint roads, twelve and eight to two group orders acting on that residence, five to a vertex degree on a different object entirely, and the refusal to harmonize them is the anti-fitting discipline running on the architecture's own structure.

The single most important open question is whether the office taxonomy is complete. Five classes cover the corpus surveyed and no argument here establishes that a sixth is impossible. A citation load-bearing on a conclusion that passes none of the four discriminator questions would settle it.

Accepting the argument entails one reframing, modest in statement and wide in reach. Depth is not a height. Any question of the form *how sophisticated is this mathematics* is answerable only after the questioner has said which of three independent quantities is being asked about. And any question of the form *does this architecture touch that mathematics* is answerable only after the questioner has said whether the word means executed, cited, or covered.

APPENDIX A: FOUNDATIONAL PRINCIPLES

The following are carried by the architecture this paper inventories, stated in standalone form so that the body's argument can be assessed without reference to the wider framework.

The actuation floor. Every physical existent carries a strictly positive energetic content. The energy floor is theorem grade for confined quantum systems on the Heisenberg kinetic-energy bound and the zero-point energy following from canonical commutation, with rest energy fixing positivity for massive systems and the rigorous third law barring attainment of a zero-energy state. The cost of any transition that occurs is charged by the fluctuation work relations, with Landauer's bound the irreversible-erasure subclass, and the rate of any transition is bounded by the quantum speed limit. The extension of the floor over all existents whatever is a premise and is held as one.

Grounding before derivation. A formal system's theorems are relative to its axioms, and the axioms are in place before any derivation exists, their being in place a condition of the sequence being a derivation at all. No derivation grounds its own starting points, by the second incompleteness theorem and the undefinability of truth. The priority is a property of the object and not a doctrine requiring adoption, and it is satisfied by every proof ever written, including those produced under other accounts of foundation.

Warrant travels with the claim. Every claim carries an explicit grade: theorem, conditional, structural, or premise. A conclusion's strength is capped at the weakest load-bearing grade in its chain. No convergence among sources raises a grade and no absence of convergence lowers one.

Three states and no fourth. Verdicts resolve to sealed, broken with a named mechanism, or open with a named violation. Fractional and probabilistic truth values are undefined in the economy. Refinements internal to the three exist; a fourth state does not.

Exhaustive routing without an escape bin. Where the architecture asserts that a field routes entirely into the three-state economy, the claim is coverage by containment and not use, citation, or origination. It is the form the architecture's no-outside argument takes against a candidate universal meta-language, and it is as strong as that containment argument and no stronger.

Authorship is held at zero. The architecture reorganizes established mathematics and claims no authorship over it. The one exit is an externally witnessed, independently verified, reproducible, adversarially audited claim of new mathematical content, and no internal reasoning however clean opens it.

APPENDIX B: AUTHOR'S PROVENANCE AND METHOD DISCLOSURE

This paper was developed under Trisduction, a verification and organizing discipline that adds the cited results no warrant. The method decomposes a claim onto three warrant axes, one of them empirical and read against measurable actuation, standardizes them, removes shared sources by orthogonal projection, and reads the determinant of what survives into a three-state verdict, with an explicit warrant grade travelling with every claim and a capping rule preventing any citation from raising a grade. The executable procedures, the screen specifications, and the recorded batteries are stated in full at the master reference in the references below. Numerical claims here are re-executable at seed 20260622 with the stream labels printed beside each sampled extremum.

Register resolved from the repository of record at branch main, mathematical corpus at version 3.22.0, theological corpus at version 1.54.0, both fetched at a pinned commit. A reader should re-resolve before citing, since the register advances.

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